

A Cosmic Battery Reconsidered

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Abstract

We revisit the problem of magnetic field generation in accretion flows onto black holes owing to the excess radiation force on electrons. This excess force may arise from the Poynting-Robertson effect. Instead of a recent claim of the generation of dynamically important magnetic fields, we establish the validity of earlier results from 1977 which show only small magnetic fields are generated. The radiative force causes the magnetic field to initially grow linearly with time. However, this linear growth holds for only a *restricted* time interval which is of the order of the accretion time of the matter. The large magnetic fields recently found result from the fact that the linear growth is unrestricted. A model of the Poynting-Robertson magnetic field generation close to the horizon of a Schwarzschild black hole is solved exactly using General Relativity, and the field is also found to be dynamically insignificant. These weak magnetic fields may however be important as seed fields for dynamos.

keywords *accretion, accretion disks—galaxies: active—plasmas—magnetic fields—stars: magnetic fields—X-rays: stars*

1 Introduction

The classical battery mechanism of magnetic field generation is connected with a noncoincidence of surfaces of constant pressure and constant density, where forces connected with pressure gradients become nonpotential or rotational. In this situation no static equilibrium in the gravitational field is possible. When considering separately the motion of electrons and ions, there is always a difference in the velocities of electrons and ions which creates electric currents and an associated magnetic field. Self-induction is very important in the battery mechanism, determining the rate of increase of the magnetic field.

Along with ion and electron pressure gradients, a nonpotential force field may arise due to the radiation force which acts predominantly on the electrons. In a spherically symmetric star, the radiation force has a potential so that no magnetic field is generated: equilibrium in the two-fluid plasma results from a distribution of an electric charge and a static radial electric field. For *geometrically-thin, optically-thick* accretion disks, Bisnovatyi-Kogan and Blinnikov (1977; hereafter BKB) showed that the radiation force above the disk has a nonpotential or rotational component. Under this condition, no electric charge distribution can give a static equilibrium. Instead, electric currents and a corresponding magnetic field are generated. The radiation forces above a thin disk gives rise to poloidal electrical current flow and a toroidal magnetic field.

In accretion flows at very low mass accretion rates, an *optically-thin, geometrically-thick* accretion flow is possible (Shapiro, Lightman, & Eardley, 1976) where the ion temperature is close to the virial temperature. In the absence of a magnetic field, and neglecting relaxation processes between electrons and ions except for binary collisions these flows are referred to as advection dominated accretion flows (ADAF) (Ichimaru 1977; Narayan & Yi 1995). In the ADAF regime the radiative efficiency of accretion may be very low, $\sim 10^3$ times less than the standard value for a geometrically thin, optically thick accretion disk. Account of processes connected with the presence of a magnetic field increases the efficiency up to at least 1/3 of the standard value (Bisnovatyi-Kogan & Lovelace 1997, 2000). Nevertheless, the disk remains geometrically thick in the optically thin regime due to high ion temperature.

Contopoulos and Kazanas (1998; hereafter CK) proposed that a cosmic battery may operate in ADAF accretion flows owing to the Poynting-Robertson (PR) effect. The PR effect acts to generate a toroidal electrical current and poloidal magnetic field. The authors found that the magnetic field may be amplified up to $\sim 10^7$ G in the vicinity of a black hole of stellar mass. Note that the PR mechanism of magnetic field generation is similar to the mechanism of BKB based on the nonpotential radiative force, where the magnetic field reached values of ≤ 10 G for a stellar mass black hole. In an optically thin disk both mechanisms act together leading to the generation of toroidal and poloidal components of the magnetic field. The influence of the PR effect on the dynamics of the surface layer of an accretion disk was treated by Mott and Lovelace (1999).

Here, we analyze the difference in conclusions between CK and BKB. The radiative force initially causes the magnetic field to grow linearly with time. However, this linear growth holds for only a *restricted* time interval which is of the order of the accretion time of the matter. In CK the interval of linear growth is unrestricted. Even though we conclude that the magnetic field due to the radiation force is weak, it may have a role as a seed field for an $\alpha - \omega$ dynamo (see for example Colgate and Li 2000).

Section 2 treats the generation of toroidal field for the case of a thin disk, while §3 treats the generation of poloidal field in an ADAF flow. Section 4 gives a General Relativistic treatment of a simplified model of PR magnetic

field generation in an accretion flow close to a Schwarzschild black hole. An appendix gives an explicit solution to the non-relativistic induction equation for the magnetic field generated by the PR effect in an ADAF flow.

2 Radiatively induced current and toroidal magnetic field production in accretion disks

Above a geometrically thin accretion disk around a black hole, the electrons are acted on by a nonpotential radiation force \mathbf{F}_L due to Thomson scattering. This was calculated by BKB,

$$\mathbf{F}_L = -R \cos \theta \nabla \phi_L = (F_{LR}, F_{L\theta}, 0) , \quad (1)$$

where a spherical coordinate system (R, θ, ϕ) is used, and ϕ_L is the “pseudopotential” of the radiation force, which may be expressed as

$$\phi_L = \frac{\sigma_T}{c} \int_{r_{in}}^{\infty} \frac{H(r) r dr}{(R^4 + r^4 + 2R^2 r^2 \cos 2\theta)^{1/2}} . \quad (2)$$

Here, the disk thickness is neglected and the cylindrical radius is $r = R \cos \theta$. The function $H(r)$ is the radiative flux emitted per unit area from one side of the disk. In the standard local accretion model,

$$H(r) = \frac{3}{8\pi} \frac{GM\dot{M}}{r^3} \mathcal{J} , \quad (3)$$

where $\mathcal{J} \equiv 1 - (r_{in}/r)^{1/2}$, and $r_{in} = 3r_S = 6GM/c^2$ is the inner radius of the disk for a non-rotating black hole of Schwarzschild radius r_S . In the disk plane, $\theta = \pi/2$, the radiative force is perpendicular to the disk,

$$F_{L\theta} = -\frac{\sigma_T}{c} H(r) = \frac{3GMm_p}{r^2} \frac{r_{in}}{r} \frac{L}{L_{Edd}} \mathcal{J} , \quad (4)$$

where

$$L = \frac{GM\dot{M}}{2r_{in}}, \quad L_{Edd} = \frac{4\pi cGMm_p}{\sigma_T} , \quad (5)$$

where σ_T is the Thomson cross section. Due to interaction of the radiation flux mainly with the electrons, the accretion disk becomes positively charged up to a value where the electrostatic attraction of the electrons balances the radiation force. The vertical component of the electrical field strength E_θ in the disk plane is written as

$$E_\theta(r) = -\frac{\sigma_T}{c|e|} H(r) = -E_0 \frac{L}{L_{Edd}} \left(\frac{r_{in}}{r} \right)^3 \mathcal{J} , \quad (6)$$

where

$$E_0(M) \equiv \frac{m_p c^4}{12|e|GM} \approx 1.76 \frac{M_\odot}{M} \text{ cgs} \approx 528 \frac{M_\odot}{M} \frac{\text{V}}{\text{cm}} \quad (7)$$

Thus, the surface charge-density of the disk is

$$\Sigma_e(r) = \frac{E_\theta(r)}{2\pi} .$$

The influence of this charge on the structure and stability of the accretion disk is negligible.

Both the gravitational and electrical forces have a potential, so that they cannot balance the nonpotential radiation force. Due to the radiation and electric forces, electrons move with respect to protons, which to a first approximation do not acquire the poloidal motion. Thus a poloidal electrical current is generated with an associated toroidal magnetic field. The finite disk thickness may create poloidal motion of all of the matter of the accretion disk, similar to meridional circulation in rotating stars (Kippenhahn & Thomas 1982). The absence of this circulation occurs for a unique dependence of the rotational velocity over the disk thickness, $\Omega(z)$.

To estimate the magnetic field strength, we write the electromotive force (EMF) as

$$\mathcal{E} = \frac{1}{e} \oint \mathbf{F}_L \times d\mathbf{l} = \frac{1}{e} \int \int d\mathbf{S} \cdot \nabla \times \mathbf{F}_L \sim E_\theta h , \quad (8)$$

where h is the half-thickness of the disk. Thus the stationary current-density is

$$J_{st} \sim \frac{\sigma_e \mathcal{E}}{r} \sim \frac{\sigma_e E_\theta h}{r} , \quad (9)$$

where σ_e is the conductivity of the disk plasma. The stationary toroidal magnetic field (BKB) is

$$B_{\phi 0} \sim \frac{4\pi}{c} J_{st} h \sim \frac{4\pi \sigma_e}{c} \frac{E_\theta h^2}{r} . \quad (10)$$

In the radiation-dominated inner region of the standard α -disk model, h can be written as

$$h = 3 \frac{L}{L_{Edd}} \mathcal{J} r_{in} , \quad (11)$$

(Shakura 1972; Shakura & Sunyaev 1973). Finally, we obtain the stationary value of toroidal magnetic field in the disk,

$$\begin{aligned} B_{\phi 0} &\sim \frac{36\pi \sigma_e}{c} E_0 \left(\frac{L}{L_{Edd}} \right)^3 \left(\frac{r_{in}}{r} \right)^4 r_{in} \mathcal{J}^3 , \\ &= \frac{12\pi \sigma_e h}{c} E_0 \left(\frac{L}{L_{Edd}} \right)^2 \left(\frac{r_{in}}{r} \right)^4 \mathcal{J}^2 . \end{aligned} \quad (12)$$

We next discuss the value of conductivity σ_e .

Bisnovatyi-Kogan and Blinnikov (1977) considered different values of the plasma conductivity σ_e , namely, the conductivity owing to binary collisions, σ_{Coul} , and the effective conductivity σ_{eff} derived by Vainshtein (1971),

$$\sigma_{Coul} \approx 3 \times 10^6 T^{3/2} s^{-1}, \quad \sigma_{eff} = \frac{\sigma_{coul}}{\sqrt{Re_{m0}}} . \quad (13)$$

The magnetic Reynolds number in a turbulent plasma is defined as

$$Re_{m0} \equiv \frac{4\pi\sigma_{coul}v_th}{c^2} , \quad (14)$$

where the turbulent velocity v_t in an α -disk is $v_t = \alpha c_s$, where $c_s = \sqrt{p/\rho}$.

In addition to the values of equation (13), we consider the conductivity in the presence of well-developed turbulence (Bisnovatyi-Kogan & Ruzmaikin 1976),

$$\sigma_{turb} = \frac{c^2}{4\pi\alpha h c_s} = \frac{\sigma_{coul}}{Re_{m0}} . \quad (15)$$

Estimations of the stationary field $B_{\phi 0}$ of equation (12) in the radiation dominated inner region of the disk around a stellar mass black hole gives values (BKB) for two cases in (13) of $B_{\phi 0} \sim 10^{13}$ G for $\sigma_e = \sigma_{Coul}$ and $B_{\phi 0} \sim 10^8$ G for $\sigma_e = \sigma_{eff}$, with $Re_{m0} \sim 3 \times 10^{10}$. For a turbulent conductivity $\sigma_e = \sigma_{turb}$, we obtain $B_{\phi 0} \sim 10$ G for $\alpha = 0.1$.

The time-scale τ_m for reaching the stationary field given by equation (12) is determined by the self-induction of the disk. This is equivalent to the “ \mathcal{L} over \mathcal{R} time” of circuit with inductance \mathcal{L} and resistance \mathcal{R} . This time-scale is equal to

$$\tau_m \simeq \frac{4\pi\sigma_e h r}{c^2} . \quad (16)$$

The crossing time of matter passing through the radiation dominated region of the disk is

$$t_c \approx \frac{r}{v_r} . \quad (17)$$

During the time t_c there is linear growth of the magnetic field after which the matter falls into the black hole. Thus the stationary value of the large scale toroidal magnetic field is

$$B_\phi \approx B_{\phi 0} \frac{t_c}{\tau_m} = 3 \frac{c}{v_r} E_0 \left(\frac{L}{L_{Edd}} \right)^2 \left(\frac{r_{in}}{r} \right)^4 \mathcal{J}^2 . \quad (18)$$

For the case of a turbulent conductivity $\sigma_e = \sigma_{turb}$, the growth time-scale of the magnetic field, using equations (15) and (16), is equal to

$$\tau_m^{turb} \approx \frac{r}{\alpha c_s} . \quad (19)$$

Taking into account that $v_r = \alpha c_s (L/L_{Edd})(r_{in}/r) < \alpha c_s$ and $t_m^{turb} < t_c$, we find

$$B_\phi = B_{\phi 0}^{turb} \approx 3 \frac{c}{\alpha c_s} E_0 \left(\frac{L}{L_{Edd}} \right) \left(\frac{r_{in}}{r} \right)^3 \mathcal{J}^2. \quad (20)$$

In that

$$c_s = 7 \times 10^9 \frac{\text{cm}}{\text{s}} \left(\frac{L}{L_{Edd}} \right) \left(\frac{r_{in}}{r} \right)^{3/2} \mathcal{J}.$$

The strength of the stationary toroidal magnetic field produced by the battery effect in the radiation-dominated region of an accretion disk with the turbulent or higher conductivity from (13) is equal to

$$B_\phi \approx \frac{22}{\alpha} \left(\frac{M_\odot}{M} \right) \left(\frac{r_{in}}{r} \right)^{3/2} \mathcal{J}. \quad (21)$$

At a distance $r = 3r_{in}$, we have

$$B_\phi \approx \frac{2}{\alpha} \left(\frac{M_\odot}{M} \right) \text{ G}. \quad (22)$$

This agrees with the findings of Bisnovatyi-Kogan and Blinnikov (1977). The corresponding magnetic energy-density is very much less than the energy-density associated with the turbulent motion in the disk $\rho v_t^2/2$.

3 Production of a poloidal magnetic field in optically thin accretion flows by Poynting-Robertson effect

In optically thin accretion flows (Shapiro, Lightman, & Eardely 1976; Ichimaru 1977; Narayan & Yi 1995), the radiation flux interacts with the inspiraling matter by the Poynting-Robertson (PR) effect (Robertson 1937). Analysis by Shakura (1972) showed that the PR effect was negligible for optically thick accretion disks. Contopoulos and Kazanas (1998) studied the PR effect as a mechanism for generating poloidal magnetic field in an optically thin accretion flow. They concluded that dynamically important magnetic field strengths could result from this effect. Here, we reconsider the PR effect for quasi-spherical advection dominated accretion flows (ADAF, Narayan & Yi 1995).

The linear growth of the magnetic field due to the radiative force on the electrons found by CK is similar to that analyzed by Bisnovatyi-Kogan and Blinnikov (1977), but the PR effect implies an additional (small) numerator, (v_ϕ/c) . Also, for a quasi-spherical accretion flow the characteristic scale is r instead of h , and the quasi-spherical luminosity is $L/(4\pi r^2)$ instead of H in (3). Then, using equations (10), (16), (18), we obtain the rate of growth the

poloidal magnetic field due to the PR effect, which is equivalent to the expression obtained by CK,

$$B_z \approx B_{z0} \frac{t}{\tau_m} = \frac{E_0}{3\alpha} \left(\frac{L}{L_{Edd}} \right) \left(\frac{r_{in}}{r} \right)^2 \left(\frac{tv_r}{r} \right). \quad (23)$$

Here, E_0 is defined in equation (6), and r_{in} in equation (3).

Now it is essential to take into account, that an element of matter with the induced magnetic field reaches the black hole in time $t_c \approx r/v_r$. (Damping of the magnetic field as it approaches the black hole horizon is discussed in Appendix A.) This means that the magnetic field grows *only* during the time t_c . Consequently, the magnetic field reaches a stationary value

$$B_z \approx \frac{E_0}{3\alpha} \frac{L}{L_{Edd}} \left(\frac{r_{in}}{r} \right)^2 \approx \frac{0.7}{\alpha} \frac{L}{L_{Edd}} \frac{M_\odot}{M} \left(\frac{r_{in}}{r} \right)^2 \quad (24)$$

Taking into account that the luminosity is $\leq 10^{-3} L_{Edd}$ for optically thin accretion, and taking $\alpha = 0.1$, we get a maximum value of the magnetic field created by the PR effect in an ADAF to a black hole,

$$B_z \approx 7 \times 10^{-3} \left(\frac{M_\odot}{M} \right) \text{ G} . \quad (25)$$

This estimate of the field is about 10 orders of magnitude less than the value obtained by CK. The difference in the estimates results from the fact that CK assume that magnetic flux accumulates continuously near the black hole during a long time, reaching the equipartition with the kinetic energy. The accumulation actually occurs only during the time the plasma (which carries the field or current loops) takes to move inward to the black hole horizon (Bisnovatyi-Kogan and Ruzmaikin, 1976). The current loops created by the PR effect disappear as the matter approaches the horizon (see Appendix A). In the case of accretion onto a neutron star or a white dwarf, matter containing the current loops merges with the stellar matter, which is typically much more strongly magnetized. After merging, the matter becomes optically thick, the action of PR effect stops, penetration of matter into the magnetosphere of the star occurs, and interaction of the accretion flux with the stellar surface takes place.

4 Magnetic field Generation in the vicinity of the Schwarzschild radius

Here we consider the Poynting-Robertson magnetic field generation on the accretion flow near a Schwarzschild black hole. The accreting matter is assumed to radially towards the black hole with velocity v_r . As before we consider the case of high conductivity matter where $t_m \gg t_c$. In a non-accreting flow, the

magnetic field can grow linearly with time as accepted by CK. As mentioned above, there is an important relativistic effect close to the black hole: Current loops in the accreting matter which approach the horizon of a black hole cannot produce a magnetic field visible by an external observer. This effect is related to the damping of the magnetic field in a collapsing star (Ginzburg & Ozernoi 1964).

The azimuthal force and the corresponding azimuthal electric field due to the Poynting-Robertson effect are

$$F_{PR} = \frac{L\sigma_T}{4\pi cr^2} \frac{v_{\phi 0}}{c} \sin \theta, \quad E_{\phi}^{(ph)} = \frac{1}{|e|} F_{\phi}^{PR}, \quad (26)$$

where

$$v_{\phi 0} = A \sqrt{\frac{GM}{r}}, \quad A \sim 1, \quad (27)$$

and where the (ph) superscript indicates the physical value of the field component. Note that in a strictly spherical accretion flow there is no azimuthal EMF. However, in the approximate model considered here the infalling matter has a small rotation so that the PR force is small and these have radial inflow only slightly.

We assume a Schwarzschild metric,

$$ds^2 = g_{00}c^2dt^2 + g_{11}dr^2 + g_{22}d\theta^2 + g_{33}d\phi^2, \quad (28)$$

where

$$g_{00} = \left(1 - \frac{r_g}{r}\right), \quad g_{11} = -\left(1 - \frac{r_g}{r}\right)^{-1}, \\ g_{22} = -r^2, \quad g_{33} = -r^2 \sin^2 \theta,$$

and $r_g \equiv 2GM/c^2$. The matter is free-falling in the radial direction with the nonzero components of a 4-velocity

$$u^0 = \left(1 - \frac{r_g}{r}\right)^{-1/2}, \quad u_0 = 1, \\ u^r = -\sqrt{\frac{r_g}{r}}, \quad u_r = \sqrt{\frac{r_g}{r}} \left(1 - \frac{r_g}{r}\right)^{-1/2}. \quad (29)$$

In any 4-dimensional time-space with metric g_{ik} (Latin indices takes values 0, 1, 2, 3), the electric E_{α} and magnetic B^{α} fields (Greek indices run over the values 1, 2, 3) in 3-space are defined through the antisymmetric electromagnetic field tensor $F_{ik} = -F_{ki}$, with zero diagonal components (Landau & Lifshitz 1988),

$$B^{\alpha} = -\frac{1}{2\sqrt{\gamma}} \varepsilon^{\alpha\beta\gamma} F_{\beta\gamma}, \quad E_{\alpha} = F_{0\alpha}, \quad (30)$$

and

$$F_{\alpha\beta} = -\sqrt{\gamma}\varepsilon_{\alpha\beta\gamma}B^\gamma ,$$

where $\varepsilon_{\alpha\beta\gamma} \equiv \varepsilon^{\alpha\beta\gamma}$ is the usual antisymmetric 3-dimensional symbol ($\varepsilon_{123} = 1$), and γ is the determinant of the 3-dimensional metric tensor, obtained by splitting the metric tensor g_{ik} into space ($\gamma_{\alpha\beta}$) and time (h) parts as

$$\gamma_{\alpha\beta} = -g_{\alpha\beta} + g_{0\alpha}g_{0\beta}/g_{00} , \quad h = g_{00} . \quad (31)$$

For the Schwarzschild metric (28), $\gamma_{\alpha\beta} = -g_{\alpha\beta}$. The first pair of Maxwell equations can be written as

$$F_{ik;l} + F_{li;k} + F_{kl;i} = 0$$

or

$$\frac{1}{c} \frac{\partial(\sqrt{\gamma}B^\alpha)}{\partial t} + \varepsilon^{\alpha\beta\gamma} \frac{\partial E_\gamma}{\partial x^\beta} = 0, \quad \frac{\partial(\sqrt{\gamma}B^\alpha)}{\partial x^\alpha} = 0 . \quad (32)$$

Note that the physical r and θ components of the magnetic field in this reference frame are $\sqrt{-g_{11}}B^r$ and $\sqrt{-g_{22}}B^\theta$ respectively. Thus the dimensions of B^θ are length $\times B^r$.

In a perfectly conducting medium moving with 4-velocity u^i , we have $F_{ik}u^k = 0$, which corresponds to a vanishing electric field in the comoving frame. This gives

$$E_\alpha = -\sqrt{-g} \varepsilon_{\alpha\beta\gamma}(v^\beta/c)B^\gamma . \quad (33)$$

In the presence of an externally imposed electric field E_α^{ext} , the electrical field E_α is

$$E_\alpha = -\sqrt{-g} \varepsilon_{\alpha\beta\gamma}(v^\beta/c)B^\gamma - E_\alpha^{ext} . \quad (34)$$

The 3-velocities v^α are given by

$$v^\alpha = \frac{c dx^\alpha}{\sqrt{h} dx^0} , \quad dx^0 = c dt , \quad (35)$$

$$u^\alpha = \frac{v^\alpha}{c} \left(1 - \frac{v^2}{c^2}\right)^{-1/2} , \quad (36)$$

where

$$u^0 = \frac{1}{\sqrt{h}} \left(1 - \frac{v^2}{c^2}\right)^{-1/2} , \quad v^2 = \gamma_{\alpha\beta}v^\alpha v^\alpha = v_\alpha v^\alpha .$$

Substituting the expression (34) into equation (32), gives the following equation for the magnetic field B^α

$$\frac{\partial(\sqrt{\gamma}B^\alpha)}{\partial t} = \frac{\partial}{\partial x^\beta} [\sqrt{-g}(B^\beta v^\alpha - B^\alpha v^\beta)] + \varepsilon^{\alpha\beta\gamma} \frac{\partial E_\gamma^{ext}}{\partial x^\beta} c . \quad (37)$$

$$\frac{\partial}{\partial x^\alpha} (\sqrt{\gamma}B^\alpha) = 0 . \quad (38)$$

For the Schwarzschild metric (28) $(x^0, x^1, x^2, x^3) = (ct, r, \theta, \phi)$, it follows from (29), (35), (36) that $v^\alpha = (v^r, 0, 0)$ and

$$v^r = c\sqrt{1 - \frac{r_g}{r}}u^r = -c\sqrt{1 - \frac{r_g}{r}}\sqrt{\frac{r_g}{r}}, \quad (39)$$

where $v^2/c^2 = r_g/r$. The value of h , the determinant g of the 4-metric tensor g_{ik} , and the determinant γ of the metric tensor $\gamma_{\alpha\beta}$ in the Schwarzschild metric are

$$h = 1 - \frac{r_g}{r}, \quad \sqrt{-g} = r^2 \sin \theta, \quad \sqrt{\gamma} = \left(1 - \frac{r_g}{r}\right)^{-1/2} r^2 \sin \theta. \quad (40)$$

With $B^\alpha = (B^r, B^\theta, 0)$ and $E_\alpha^{ext} = (0, 0, E_\phi^{ext})$, and with all quantities independent on the azimuthal angle ϕ , equations (37) and (38) give

$$\frac{\partial(\sqrt{\gamma}B^r)}{\partial t} + \sqrt{h}v^r \frac{\partial(\sqrt{\gamma}B^r)}{\partial r} = c \frac{\partial E_\phi^{ext}}{\partial \theta}, \quad (41)$$

$$\frac{\partial(\sqrt{-g}v^r B^\theta)}{\partial t} + \sqrt{h}v^r \frac{\partial(\sqrt{-g}v^r B^\theta)}{\partial r} = -c v^r \sqrt{h} \frac{\partial E_\phi^{ext}}{\partial r}, \quad (42)$$

$$\frac{\partial}{\partial r}(\sqrt{\gamma}B^r) + \frac{\partial}{\partial \theta}(\sqrt{\gamma}B^\theta) = 0. \quad (43)$$

Equations (41) and (42) with known right-hand-sides are solved using the characteristic method.

The integrals of the characteristic equations can be written as

$$t - \int_{r_0}^r \frac{dr}{\sqrt{h}v^r} = C_1, \quad (44)$$

$$\sqrt{\gamma}B^r - c \int_{r_0}^r \frac{\partial E_\phi^{ext}}{\partial \theta} \frac{dr}{v^r \sqrt{h}} = C_2, \quad (45)$$

$$\sqrt{-g}v^r B^\theta + c \int_{r_0}^r \frac{\partial E_\phi^{ext}}{\partial r} dr = C_3. \quad (46)$$

The constants C_i are determined by the initial conditions. For the present problem these are

$$B^r = B^\theta = 0, \quad r = r_0 \text{ at } t = 0,$$

which implies $C_i = 0$. In the general case the constants C_2, C_3 are determined by the initial values of B^r, B^θ , which should satisfy zero divergence condition (43). We may in general take $C_1 = 0$, fixing the reference frame $r = r_0$ at $t = 0$. With account of equations (39) and (40), equations (45) and (46) can be written as

$$B^r = -\frac{\sqrt{r - r_g}}{r^{5/2} \sqrt{r_g} \sin \theta} \frac{\partial}{\partial \theta} \left[\int_{r_0}^r E_\phi^{ext} \frac{r^{3/2} dr}{r - r_g} \right], \quad (47)$$

$$B^\theta = \frac{1}{r\sqrt{r-r_g}\sqrt{r_g}\sin\theta} [E_\phi^{ext}(r, \theta) - E_\phi^{ext}(r_0, \theta)] . \quad (48)$$

Integration of equation (44) with $C_1 = 0$ gives a relation between t , r and r_0 in the form

$$\begin{aligned} \frac{ct}{r_g} + \frac{2}{3}x^{3/2} + 2x^{1/2} + \ln \frac{\sqrt{x}-1}{\sqrt{x}+1} \\ = \frac{2}{3}x_0^{3/2} + 2x_0^{1/2} + \ln \frac{\sqrt{x_0}-1}{\sqrt{x_0}+1} , \end{aligned} \quad (49)$$

where $x = r/r_g$, and $x_0 = r_0/r_g$ (Bisnovaty-Kogan and Ruzmaikin, 1974).

Consider now the Poynting-Robertson EMF, $E_\phi^{ext}(r, \theta)$. First we show that E_ϕ^{ext} must tend to zero as $r \rightarrow r_g$ at least as fast as $(r - r_g)$ or faster in order to avoid singularity at r_g . This means that in the comoving reference system with metric

$$ds^2 = c^2 d\tau^2 - \frac{r_g}{r} d\rho^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) , \quad (50)$$

there is no singularity at the black hole horizon. The connection between Schwarzschild and comoving coordinates (τ, ρ) (the angle coordinates θ and ϕ are the same) is

$$\begin{aligned} c\tau &= ct + r_g \left[2\sqrt{x} + \ln \frac{\sqrt{x}-1}{\sqrt{x}+1} \right] , \\ \rho &= ct + r_g \left[\frac{2}{3}x^{3/2} + 2x^{1/2} + \ln \frac{\sqrt{x}-1}{\sqrt{x}+1} \right] \end{aligned} \quad (51)$$

We can now connect the magnetic field in a comoving system $\dot{B}^\alpha = (\dot{B}^\rho, \dot{B}^\theta, 0)$ with the field in the Schwarzschild system in terms of Schwarzschild variables (r, t) as

$$\begin{aligned} \dot{B}^\rho &= \frac{r}{\sqrt{r-r_g}\sqrt{r_g}} B^r , \\ \dot{B}^\theta &= \frac{\sqrt{r-r_g}}{\sqrt{r}} B^\theta + \sqrt{\frac{r_g}{r}} \frac{E_\phi^{ext}}{r(r-r_g)\sin\theta} . \end{aligned} \quad (52)$$

It follows from equations (47), (48), and (52) that there is no singularity in the comoving frame if E_ϕ^{ext} tends to zero as $(r - r_g)$ or faster as $r \rightarrow r_g$. The metric tensor in this system (50) is regular on the horizon, so with finite \dot{B}^α all 4-invariants (for example, $F_{ik}F^{ik}$) are also regular there. In fact, we can obtain the dependence of E_ϕ^{ext} from equation (26), taking into account that the covariant component E_ϕ^{ext} in equations (41) and (42) is connected with the physical component $E_\phi^{(ph)}$ from equation (26) as

$$E_\phi^{ext} = \sqrt{\gamma_{\phi\phi}} E_\phi^{(ph)} = r \sin \theta E_\phi^{(ph)} . \quad (53)$$

The luminosity L seen by a distant observer viewing collapsing matter with constant comoving luminosity L_0 is

$$L = L_0 \left(1 - \frac{r_g}{r}\right)^4, \quad (54)$$

(Zeldovich & Novikov 1971). Thus we have from equations (26), (27), (53), and (54)

$$E_\phi^{ext} = Dr^{-3/2} \left(1 - \frac{r_g}{r}\right)^4 \sin^2 \theta, \quad (55)$$

where

$$D = \frac{L_0 \sigma T A \sqrt{GM}}{4\pi c^2 |e|}. \quad (56)$$

Equation (55) is of course simplified but the dependence allows an estimate to be made of the magnetic field generation by the PR effect close to a black hole. It is necessary that $E_\phi^{(ph)}$ vanish sufficiently rapidly on the horizon in order to avoid a physical singularity, but the exact dependence is not important.

Substituting equation (55) into (41) and (42) gives

$$B^r = \frac{2D}{\sqrt{r_g}} \frac{\sqrt{r-r_g}}{r^{5/2}} \left[\ln \frac{r_0}{r} - \frac{1}{3} \left(\frac{r_g}{r}\right)^3 + \frac{3}{2} \left(\frac{r_g}{r}\right)^2 - 3\frac{r_g}{r} + \frac{1}{3} \left(\frac{r_g}{r_0}\right)^3 - \frac{3}{2} \left(\frac{r_g}{r_0}\right)^2 + 3\frac{r_g}{r_0} \right] \cos \theta, \quad (57)$$

$$B^\theta = \frac{D}{\sqrt{r_g} r \sqrt{r-r_g}} \left[\frac{1}{r^{3/2}} \left(1 - \frac{r_g}{r}\right)^4 - \frac{1}{r_0^{3/2}} \left(1 - \frac{r_g}{r_0}\right)^4 \right] \sin \theta. \quad (58)$$

There are several limiting cases where expressions for B^r and B^θ can be written in a simpler form.

1. **Non-Relativistic, Newtonian Regime:** Here, $r, r_0 \gg r_g$, and from equation (49) we have

$$x_0 = x \left(1 + \frac{3}{2x^{3/2}} \frac{ct}{r_g}\right)^{2/3}, \quad (59)$$

and from equations (57) and (58),

$$B^r = \frac{4D \cos \theta}{3r^2 \sqrt{r_g}} \ln \left(1 + \frac{3}{2x^{3/2}} \frac{ct}{r_g}\right) \quad (60)$$

$$B^\theta = \frac{D \sin \theta}{r^3 \sqrt{r_g}} \left[1 - \left(1 + \frac{3}{2x^{3/2}} \frac{ct}{r_g}\right)^{-1} \right]. \quad (61)$$

We see here that for large t the physical value rB^θ tends to a finite limit of the order of equation (24), while B^r grows but only logarithmically. During the accretion time to a massive black hole this logarithm does not exceed ~ 25 . Thus the magnetic field is larger by a factor ~ 25 than the estimations of equations (24) and (25). Still, the magnetic field is enormously less than the value found by CK.

2. Vicinity of the Gravitational Radius: Here, $(x-1) \ll 1$ and we have from equation (49)

$$x-1 = 4 \frac{x_0-1}{(\sqrt{x_0}+1)^2} \exp\left(-\frac{ct}{r_g} - \frac{8}{3} + \frac{2}{3}x_0^{3/2} + 2x_0^{1/2}\right). \quad (62)$$

For matter initially situated in the vicinity of the horizon the Lagrangian coordinate $x_0-1 \ll 1$. In this case we have

$$\ln \frac{x_0-1}{x-1} = \frac{ct}{r_g}. \quad (63)$$

From equation (57),

$$B^r = \frac{D \cos \theta}{2 r_g^{5/2}} (x-1)^{9/2} [\exp(4ct/r_g) - 1]. \quad (64)$$

From equation (58),

$$B^\theta = -\frac{D \sin \theta}{r_g^{7/2}} (x-1)^{7/2} [\exp(4ct/r_g) - 1]. \quad (65)$$

These relations are valid also at large t while $(x-1)e^{ct/r_g} \ll 1$.

For matter with an intermediate Lagrangian coordinate, $(x_0-1) \sim 1$, the vicinity of the horizon r_g is reached only at very large t , so that

$$B^r = 2D \cos \theta \frac{\sqrt{x-1}}{r_g^{5/2}} \left(\ln x_0 - \frac{11}{6} + \frac{1}{3x_0^3} - \frac{3}{2x_0^2} + \frac{3}{x_0} \right), \quad (66)$$

$$B^\theta = -\frac{D \sin \theta}{r_g^{7/2} \sqrt{x-1}} \frac{(x_0-1)^4}{x_0^{11/2}}. \quad (67)$$

We see that in the vicinity of the horizon $(x-1) \ll 1$ for matter with an intermediate x_0 , $(x_0-1) \sim 1$, the component B^θ in the Schwarzschild coordinates diverges. However, this is a natural coordinate singularity which means only that a Schwarzschild observer cannot exist physically in this region. Consequently, the magnetic field in Schwarzschild coordinates has observable consequences only in the region $x \gg 1$, $r \gg r_g$.

Matter at a very large Lagrangian radius reaches the vicinity of the gravitational radius at very late times. Consider a case with

$$x_0-1 \gg x, \quad x-1 \ll 1. \quad (68)$$

Here, for very large $ct/r_g \gg |\ln(x-1)|$ we have $x_0 = (3ct/2r_g)^{2/3}$ and

$$B^r = \frac{4D \cos \theta}{3r_g^{5/2}} (x-1)^{1/2} \ln \left(\frac{3ct}{2r_g} \right), \quad (69)$$

If we assume, in addition that $x_0 \gg (x-1)^{-8/3}$, we obtain

$$B^\theta = \frac{D \sin \theta}{r_g^{7/2}} (x-1)^{7/2}. \quad (70)$$

From a comparison of the asymptotic relations (64) - (69), we see that for r approaching r_g , the Schwarzschild component B^r grows exponentially with time very close to the horizon while remaining zero at the horizon. For larger r it grows logarithmically which is the dependence in the Newtonian domain. The Schwarzschild component B^θ has the mentioned singularity on the horizon, which is unobservable because a physically realizable observer cannot measure it. However, at any fixed value of r close enough to r_g , the temporal behavior of the B^θ can be obtained from equations (65), (67), and (70). In this region B^θ starts to grow exponentially with time, but at long times it tends to a constant value. This component rapidly decreases with increasing r .

It is of interest to determine the magnetic field “seen” by an observer located far from the black hole who measures the field near the gravitational radius by means of the cyclotron radiation coming from this region. The observer determines the field strength by measuring the cyclotron frequency of the radiation. The frequency of the emitted radiation is determined by the comoving field strength, that is, by \dot{B}^ρ and \dot{B}^θ from (52) evaluated in the vicinity of r_g . We now obtain estimates of these fields. It follows from equation (62) that matter with Lagrangian coordinate x_0 approaches the horizon as $t \rightarrow \infty$. The coordinate x_0 parametrizes the horizon points and the radial coordinate x parametrizes points of the initial hypersphere $t = 0$. From equations (52), (57), (58), (66), and (67) we obtain the comoving fields in this region,

$$\dot{B}^\rho = \frac{2D \cos \theta}{r_g^{5/2}} \left(\ln x_0 - \frac{11}{6} + \frac{1}{3x_0^3} - \frac{3}{2x_0^2} + \frac{3}{x_0} \right), \quad (71)$$

$$\dot{B}^\theta = -\frac{D \sin \theta}{r_g^{7/2}} \frac{(x_0 - 1)^4}{x_0^{11/2}}. \quad (72)$$

It is easy to verify, that for any value of r_0 , $r_g < r_0 < \infty$, we have

$$\dot{B}^\rho < \frac{2D |\cos \theta|}{r_g^{5/2}} \ln x_0. \quad (73)$$

For accretion to a stellar mass or massive black hole the logarithmic factor does not exceed ~ 25 , and the value of \dot{B}^ρ remains of the same order of magnitude

as the Schwarzschild component B^r in the Newtonian region, equation (60) at large times, formally extrapolated to $r \sim r_g$. The poloidal component \dot{B}^θ , given by equation (72), is equal to zero at $r_0 = r_g$, and $r_0 = \infty$, and has a maximum at $r = 11r_g/3$. Thus

$$\dot{B}^\theta < |\dot{B}^\theta|_{max} = \frac{\lambda D \sin \theta}{r_g^{7/2}}, \quad (74)$$

where $\lambda = (3^3 8^8 11^{-11})^{1/2} \approx 0.04$. Therefore, the possible values of \dot{B}^θ are less than the Newtonian value of the Schwarzschild component B^θ (61) at large time near r_g .

The proper cyclotron frequency ω_0 of radiation emitted at (r, t) , but measured with respect to the comoving time is

$$\omega_0 = \frac{|e|\hbar}{mc} \left[\sqrt{\frac{r_g}{r}} (\dot{B}^r)^2 + r^2 (\dot{B}^\theta)^2 \right]^{1/2}, \quad (75)$$

where m is the electron rest mass. Using equations (73) and (74), we obtain an estimate for the upper limit on this frequency,

$$(\omega_0)_{max} \sim \frac{D|e|\sqrt{2}}{mcr_g^{5/2}} \sqrt{\ln^2 x_0 + \frac{\lambda^2}{4}}. \quad (76)$$

The frequency measured by a distant observer ω is related to ω_0 as

$$\omega = \omega_0 \sqrt{h} \frac{\sqrt{1 - v^2/c^2}}{1 - v \cos \psi / c}, \quad (77)$$

where $v/c = \sqrt{r_g/r}$ and ψ is the angle between v^α (which is in the negative r -direction) and the direction of the photon trajectory in Schwarzschild coordinates. For a radially emitted photon ($\psi = \pi$) we have from equation (77), $\omega = \omega_0(1 - \sqrt{r_g/r})$ near the horizon, and for the tangential direction ($\psi = \pi/2$) we have $\omega = \omega_0(1 - r_g/r)$. Due to the upper limit on ω_0 , a distant observer does not see the light emitted very close to the horizon, because of the very large red shift. Thus for a distant observer the relativistic region close to the horizon is unobservable.

Thus we conclude that the magnetic field produced by the PR effect close to the horizon of a black hole can be safely estimated using the Newtonian approximation given by equations (60) and (61). The estimated magnetic fields are dynamically insignificant.

5 Conclusion

We have reconsidered the battery effect in accretion flows due to the non-potential nature of the radiation force on the electrons. We considered cases of a

geometrically-thin, optically-thick disk where a toroidal magnetic field is generated, and a geometrically-thick, optically-thin ADAF type accretion flow where a poloidal magnetic field is generated due to the Poynting-Robertson effect. For a stellar mass black hole the generated toroidal field is estimated to be ≤ 10 G, while the poloidal field in an ADAF flow is ≤ 0.01 G. The fields vary inversely with the black hole mass. In both cases the fields are dynamically insignificant. The very large fields obtained by CK resulted from assuming unrestricted linear growth of the magnetic field. The field grows only during the accretion time. A General Relativistic treatment of the Poynting-Robertson generated magnetic field close to the horizon of a black hole shows that the field magnitude may be larger by a factor ≤ 25 than the values obtained with a non-relativistic treatment. Even though the magnetic field due to the radiation force is weak, it may have a role as a seed field for an $\alpha - \omega$ dynamo (see for example Colgate and Li 2000).

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Appendix A: Equation for Poloidal Magnetic Field Due to PR Effect

Here, we treat in more detail the influence of the azimuthal radiation force F_ϕ^{PR} which is *rotational*, and it *cannot* be balanced by any axisymmetric electrostatic field. The radiation is mainly from the central region of the flow so that the radiation flux-density is $S \approx L/(4\pi R^2)$, with L the accretion luminosity, and R the distance from the origin. The PR radiation force on an electron is $F_\phi^{PR} = -S\sigma_T v_\phi/c^2$, where σ_T is the Thompson cross section and v_ϕ is the azimuthal velocity of the accreting matter. Including the radiation force, Ohm's for the plasma is

$$\mathbf{J} = \sigma_e (\mathbf{E}^{PR} + \mathbf{E} + \mathbf{v} \times \mathbf{B}/c) , \quad (78)$$

where σ_e is the electrical conductivity, and $\mathbf{E}^{PR} = \hat{\phi} S\sigma_T v_\phi/(|e|c^2)$ is the Poynting-Robertson electric field.

Combining Faraday's and Ampère's laws and equation (78) gives

$$\frac{d\Psi}{dt} \equiv \frac{\partial\Psi}{\partial t} + \mathbf{v} \cdot \nabla\Psi = crE_\phi^{PR} + \eta_e \Delta^* \Psi , \quad (79)$$

where $\eta_e = c^2/(4\pi\sigma_e)$ is the magnetic diffusivity, and $\Psi = rA_\phi$ is the flux function and A_ϕ is the toroidal component of the vector potential. Also, $\Delta^* =$

$\partial^2/\partial r^2 - (1/r)\partial/\partial r + \partial^2/\partial z^2$ in cylindrical coordinates and $\Delta^* = \partial^2/\partial R^2 + [(1 - \mu^2)/R^2]\partial^2/\partial \mu^2$ in spherical coordinates where $\mu = \cos \theta$. Note that $B_r = -(1/r)\partial\Psi/\partial z$ and $B_z = (1/r)\partial\Psi/\partial r$ in cylindrical coordinates, while $B_R = (R^2 \sin \theta)^{-1}\partial\Psi/\partial \theta$, and $B_\theta = -(R \sin \theta)^{-1}\partial\Psi/\partial R$ in spherical coordinates. Taking $v_\phi = (GM/R)^{1/2}g(\theta)$, with $g(\pi/2) = 1$ and $g(\theta \rightarrow 0, \pi) = 0$, we find

$$E_\phi^{PR} = \frac{m_p c^2 g}{6^{3/2}|e|} \frac{L}{L_{Edd}} \frac{r_{in}^{3/2}}{R^{5/2}} \approx \frac{2gE_0}{6^{3/2}} \frac{L}{L_{Edd}} \left(\frac{r_{in}}{R}\right)^{5/2}, \quad (80)$$

where $E_0(M)$ is given by equation (7). Equation (79) is equivalent to equation (8) of CK.

For an ADAF flow, the poloidal velocity is $v_R = -\alpha\xi(GM/R)^{1/2}$ where α is the Shakura-Sunyaev parameter and $\xi \leq 1$ is a constant (Narayan & Yi 1995). Following Contopoulos and Kazanas we write $\eta_e = \mathcal{P}R|v_R|$, where \mathcal{P} , the magnetic Prandtl number, is the ratio of magnetic diffusivity to viscosity. Measuring R in units of r_{in} and t in units of $t_0 = r_{in}^{3/2}/(\alpha\xi\sqrt{GM}) = \sqrt{6}(r_{in}/c)/(\alpha\xi)$, equation (80) becomes

$$\frac{\partial\Psi}{\partial t} = \frac{Kg(\theta)\sin\theta}{R^{3/2}} + \frac{1}{\sqrt{R}}\frac{\partial\Psi}{\partial R} + \mathcal{P}\sqrt{R}\left(\frac{\partial^2\Psi}{\partial R^2} + \frac{1-\mu^2}{R^2}\frac{\partial^2\Psi}{\partial \mu^2}\right), \quad (81)$$

where $K \equiv r_{in}^2 E_0(L/L_{Edd})/(3\alpha\xi)$.

The time-scale of the linear growth of Ψ is $\tau_m = t_0/\mathcal{P}$. For a turbulent magnetic diffusivity where $\mathcal{P} = \mathcal{O}(1)$, this time-scale is quite short, $\sim t_0 \approx 7.3 \times 10^{-4} \text{s} (M/M_\odot)(0.1/\alpha)(1/\xi)$. Therefore, the physically relevant solution to equation (81) is the stationary one where the Poynting-Roberston term $\propto K$ is balanced by diffusion. This gives $Kg(\theta)\sin\theta = -\mathcal{P}(1-\mu^2)\partial^2\Psi/\partial \mu^2$ so that Ψ is independent of R . For example, for $g(\theta) = \sin(\theta)$, $B_R = (K/\mathcal{P})\cos\theta/R^2$ or

$$B_R^{PR} \approx \frac{0.6 \cos \theta}{\alpha} \frac{L}{\xi \mathcal{P}} \frac{L}{L_{Edd}} \frac{M}{M_\odot} \left(\frac{r_{in}}{R}\right)^2 \text{ G}, \quad (82)$$

and $B_\theta = 0$. This estimate agrees with equation (24). Equation (82) corresponds to a radially outward field in the northern hemisphere and a radially inward field in the southern hemisphere. The polarity of the field agrees with the PR drag on the electrons in the $-\hat{\phi}$ giving a ring current in the $+\hat{\phi}$ direction, while the simple nature of the field results from the approximation that $S \approx L/(4\pi R^2)$.

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